



CREATIVITY IN MATHEMATICS: CAN WE DEFINE, ASSESS AND DEVELOP IT IN PUPILS

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Word Count (excl. references): 4528 in total (part a& b 1779 part c 2749)

Problem and Reflection Part a & b

When I first received the assignment about investigating a “new-for-you” problem of Mathematics, I found it quite difficult to focus on a specific topic. After teaching Mathematics for over 25 years, I have encountered and taught most of the modules at A Level and University admissions papers. However, at the same time, I was beginning to teach this year’s Oxbridge candidates at my school and hence I chose to look at the MAT, STEP and UKMT challenge questions. The clear difference from those and examination papers at GCSE and A Level is the actual topic being assessed is not always clear and often a combination of topics in the field. Although all of them were challenging I found the MAT and STEP had, on the whole a clear pathway or scaffolding to the answer whilst the UKMT did not. In the end, after I had done a number of questions, I decided on UKMT 2018 Question 25

25. A semicircle is inscribed in a quarter circle as shown.

What fraction of the quarter circle is shaded?

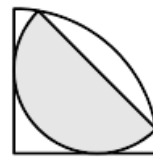
A $\frac{1}{3}$

B $\frac{1}{\sqrt{3}}$

C $\frac{2}{3}$

D $\frac{\sqrt{3}}{2}$

E $\frac{1}{\sqrt{2}}$



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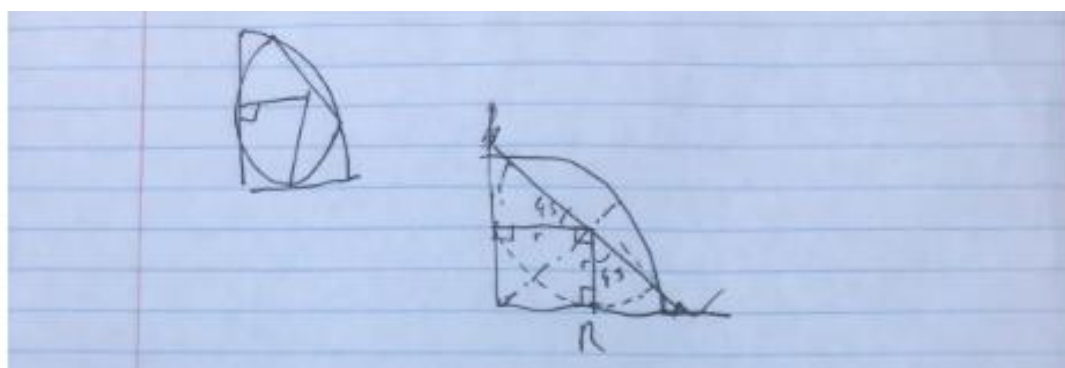
www.ukmt.org.uk

I chose this question for a number of reasons. Firstly, in my previous school the Maths teachers always had a competition to complete the paper whilst they were invigilating. I can still remember the nerves from the experience and this helped me to understand the pupils’ own feelings after an examination. Secondly, I dislike these type of geometry questions as I feel that it is my weakest area of Mathematics. One of the reasons for this is that I feel there is a pathway or single solution and if you cannot spot it then you cannot proceed on with the question. In my own head, there is a “trick” to be found or known. One strategy from my teaching experience is that I skim read a chapter to see the method, and then apply it to the questions but with the UKMT this is impossible. Thirdly, these questions are part of the United Kingdom Maths Challenge for pupils aged 16 – 18 where they are assessing their ability or maybe better to say giftedness in this subject. As someone who has a strong understanding of Mathematics, I become annoyed when I cannot answer a

question. I think this is the result of believing there is one process to answering the question, which I have just not met before and hence the question, probably wrongly, in my mind is “unfair”.

Another reason for choosing this question was the marking of it. As can be seen from the front cover, the Senior Maths Challenge has 25 multiple-choice questions. Each question is worth four marks and a wrong answer receives -1 mark. The questions become more challenging as you go process through them. Approximately, you would have 5 minutes on this question, as it is the last one. I question if this is the best way to assess giftedness and creativity in Mathematics.

Answering the Question



My first step when answering this question was to draw the diagram out (I have met these types of problems before and drawing on the real diagram and then realising something is wrong means you obscure the original). This is something that I tell my pupils to do as it can actually give you a “feel” for the problem. Straight away, I realised that my diagram looked awful (in the UKMT a candidate cannot have a ruler) but I had spotted that the semi-circle touches the quarter circle at 90° . After I had drawn my second larger version, I thought that there was a square in the bottom left hand corner. I say “thought” as the diagram might have misled me to this conclusion. After drawing out the second diagram, I then assigned the larger quarter circle with radius R and the smaller semi-circle radius r . I also added in the 45° but I did wonder if this was correct. With these type of questions I find both pupils and myself often assume certain things based on it “looking like it” in the diagram. As a reflection, if I were teaching a method of answering this question, I would be using labels for the points.

I knew from the question that we had to work out the ratio of the two areas, which I have recorded.

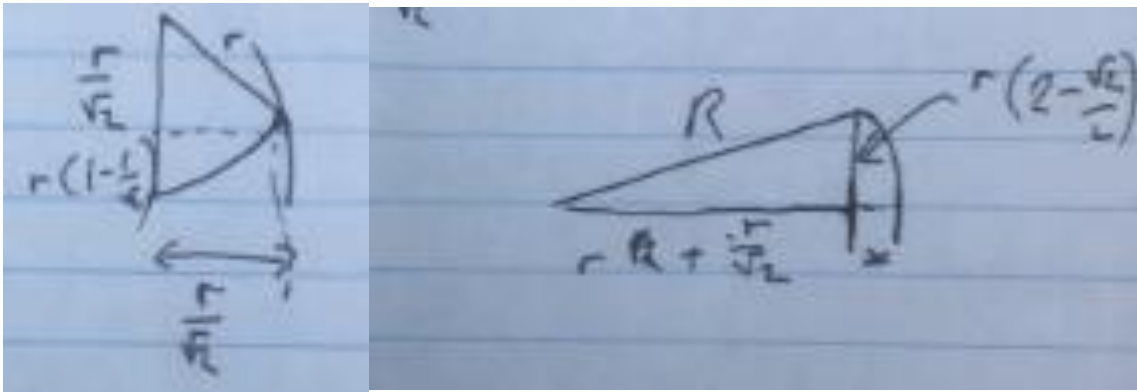
The image shows a piece of lined paper with handwritten mathematical equations. The first equation is $\frac{\sqrt{2}R^2}{4} = \frac{\sqrt{2}r^2}{2}$. The second equation is $R^2 = 2r^2$.

However, this was just a dead end as was continuing the diameter line of the semicircle (which is why it is crossed out on the above example). One aspect I thought about as we had 45° was the fact that we could utilise that $\sin 45^\circ$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$

I then investigated whether I could create an equation that linked the two radii, R and r. I did give this some thought but the usual worries of not being able to see the way the questioner wanted me to see it entered my mind. In some questions, I feel there is a simple way of seeing the solution and if you could just follow the reasoning and process, you would solve it. However, I felt I could not see this pathway and was stuck on how to move forward, something that I know happens to students I have taught as well. Often, with these style of questions I end up with trying to link two “quantities” and end up, as I did in this case, with $R=r$ a little less than r.

By this point, I was past my allocated five-minute mark as explained earlier.

I then did something new (which a student would not have time in an examination to do) I stopped and reflected. I looked at different ways and relationships between the various lines. More importantly, I felt it did not matter if I got it right or wrong (honestly I could just pick another question to attempt) and I thought about other things unconnected with the question (I was listening to some music in my study with the sun shining). Suddenly, it came to me. I could see that I could make a relationship between the two radii if I dropped the line down. This would create a right angle and using the $\sin 45^\circ$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$ as outlined earlier, I could work out the third side as well.



There are two things to highlight here about my working. Firstly in the rush to write down the third length I initially had it wrongly written on the far right hand side (I have $r\left(2 - \frac{\sqrt{2}}{2}\right)$ when the denominator should be across both numbers. I correct this straight away in my working below. Secondly, the triangle I then make below to use Pythagoras' Theorem is only clear in my head, it is not marked anywhere on my paper. I mention both these points as if someone was to assess this it would be difficult for them to follow and check my work. In this case, it does not matter as you only give a final answer and I was doing it for myself but it may be something I will need to consider later.

I had now created a right-angled triangle and could use Pythagoras' Theorem to create a relationship between R and r.

$$R^2 = r^2 \left(1 + \frac{1}{\sqrt{2}}\right)^2 + r^2 \left(1 - \frac{1}{\sqrt{2}}\right)^2$$

$$R^2 = r^2 \left(1 + \frac{2}{\sqrt{2}} + \frac{1}{2}\right) + r^2 \left(1 - \frac{2}{\sqrt{2}} + \frac{1}{2}\right)$$

$$R^2 = 3r^2$$

Now with this relationship could find the ratio of the areas

$$= \frac{\frac{1}{2}\pi r^2}{\frac{1}{4}\pi R^2} = \frac{\frac{1}{2}\pi r^2}{\frac{1}{4}\pi 3r^2} = \frac{2}{3}$$

Therefore, now the moment had arrived, the scary bit (even for a teacher) where you hope that your answer is at least one of the answers on the sheet, which it was. I then checked the official answers and I was correct! Looking at my answer sheet (appendix A) later, I admit being slightly disturbed with the presentation. There is not a formal process, more ideas and thoughts, with the answer seemingly coming organically out at the end. To be honest, I was pleased and relieved. It was rewarding to be in the learning pit and climb out of it. Previously, as a member of the schools SLT, I was often busy at work with meetings when not teaching which did not give me time to reflect on these questions. I think it is important for my pupils to witness me working out solutions rather than presenting them as a “fait de complis”. Hence they may feel they are meant to understand and be able to do every question immediately. I enjoyed times when I would go away not being able to do a problem and the answer would just pop into my head later (sometimes during the night even). However, my sense of joy was slightly knocked back as included in the answers are the “official” solutions.

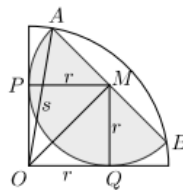
SOLUTION C

Let O be the centre of the quarter circle, A and B be the ends of the diameter of the semicircle and M be the centre of the semicircle, and hence the midpoint of AB .

Let P and Q be the points on the straight edges of the quarter circle where the quarter circle is tangent to the semicircle.

We let the radius of the quarter circle be s and let the radius of the semicircle be r .

Then $OA = s$.



As a tangent to a circle is perpendicular to its radius, $\angle OPM = \angle OQM = 90^\circ$. Because also $\angle POQ = 90^\circ$, all the angles of the quadrilateral $OQMP$ are right angles. Also $PM = r = QM$. Hence $OPMQ$ is a square with side length r .

Applying Pythagoras' theorem to the right-angled triangle OQM gives $OM^2 = r^2 + r^2 = 2r^2$.

The triangle AMO has a right angle at M . Therefore, by Pythagoras' theorem applied to this triangle, we have

$$AO^2 = OM^2 + MA^2$$

and hence

$$\begin{aligned} s^2 &= 2r^2 + r^2 \\ &= 3r^2. \end{aligned}$$

The area of the quarter circle is $\frac{1}{4}\pi s^2$. The area of the shaded semicircle is $\frac{1}{2}\pi r^2$.

Therefore, the fraction of the quarter circle which is shaded is given by

$$\frac{\frac{1}{2}\pi r^2}{\frac{1}{4}\pi s^2} = \frac{2r^2}{s^2} = \frac{2r^2}{3r^2} = \frac{2}{3}.$$

Although there is obviously a similar thread in the method, I felt that the official solutions were more elegant and precise.

Final Reflections

Was their method “better” than mine was; was it more elegant or creative? What do we mean by those terms, especially creative?

I still remember, over fifteen years ago, seeing one of the most creative moments in my teaching career. I was reviewing with a pupil the proof of $\sqrt{2}$ being irrational when we got to the stage $2b^2 = a^2$. Normally, at this point, we discuss the fact that this means a^2 is even and continue with the proof in a way known to many Mathematicians. However, this pupil stopped for a while, looked at it, and then said, “Well that’s a contradiction”. What he had spotted was to do with the Uniqueness of Prime Factorisation and that one of the squares therefore had an odd amount of prime factors, an impossibility. Not only did he demonstrate originality he could see an interconnectedness in Mathematics showing creativeness.

When we talk about giftedness or creativity in Mathematics, can we actually define it? In the process of creating the answer above I definitely felt there was a moment when it all suddenly came together, I could see the pathway to the answer. Surprisingly that moment came when I stopped thinking earnestly about the question. However, in a lesson or test is there time for this? I remember as a pupil myself that often the solution path to an A Level question used to come to me after I had finished my Maths lesson. Is this a natural part of the process?

The UKMT is thought to be a way of assessing the best Mathematicians in the country but is that possible in this timed way? Can creativity be assessed in a paper and pencil method at all? Does the attainment process the ability to create different ways to an answer important?

Finally, if we can define creativity, then can we help to develop it in our pupils? Looking at my own solution, this took time, and a sense of detachment. Can we help to encourage this whilst in a curriculum that is time burdened with timed assessments? Alternatively, would developing their creativity also improve a pupil’s overall attainment in Mathematics?

Creativity in Mathematics: Can we define, assess and develop it in pupils

Introduction

Creativity in Mathematics can recall one of the much-quoted phrase that it cannot be described but “but I know it when I see it”. If we are to define and assess it, there are a number of areas that we need to consider. In order to incorporate creativity more into the teaching experience we will need to look at the creative process, the approach to studying creativity, and finally the environment where creativity can occur. Although the definition of creativity starts with the absolute of work on the very cutting edge of Mathematics, we will define the relative creativity that a pupil could produce in the classroom.

Defining Mathematical Creativity

It seems that any characterisation of creativity needs to start with Mann (2006). Stating there was a “lack of an accepted definition for mathematical creativity” (p 3) he mentions there were over 100 contemporary ones. First, we need to look at some of the definitions of the creative process itself.

In the early part of the 20th century, the Gestalt movement defined the creative process in terms of four stages (Sriraman 2004 citing Wallas 1926, Poincaré 1945, Hadamard 1945), namely: preparation, incubation, illumination and verification. The preparation stage is a process of fully understanding the problem being posed. At University level, this may include the use of heuristics and researching thoroughly before starting in a new field. At school level, we talk of understanding what the task is asking and what information can be gleaned. The next stage of the Gestalt model, the incubation, is when time is given and the problem may even be put aside for the subconscious to develop the answer. In examples given in Srirman (2004) using conversations with University Professors, they had both time and other work to occupy themselves. Hence, the answer would be formed first on a subconscious level leading to the illumination of the third stage. The final stage of verification concerns checking and explaining that the answer is correct, something I have found pupils do not carry out on a regular basis. Considering a pupil in the classroom, my own experience would say that the incubation stage happens infrequently in a learning environment. So creative acts should be planned for more as they occur less commonly at school.

In comparison to the four-stage Gestalt model of the creative process, Ervynck (1991) described a three-stage model of preliminary technical stage (described by him as assembling the toolkit); algorithmic activity (using the toolkit) and creative activity (the conceptual moment) and a number of similarities to the Gestalt model are obvious. The early stages are an understanding of the problem and a carrying out of known algorithms. Although time is not mentioned with Ervynck, there is a transition to the final stage where the creative process occurs.

This is the point where a choice is made and deduction happens. Poincaré (1948) also mentions the use of choosing but how much choice is a pupil allowed in a normal lesson? In teaching is it common to ask our pupils to “assemble the toolkit?” More likely is the fact that the teacher will show the single algorithmic process they wish the pupils to carry out in a particular lesson. To push Eryvynck’s metaphor, it is as if the pupil looks into the toolkit and before they can decide upon which one to use for the job, the teacher swoops in, pulls out a tool and shuts the toolbox behind them. This may be said to be the “bottom line teaching” described by Crosswhite (p 268 1987) where the pupil and teacher play a waiting game until the teacher ends up telling them which algorithmic process they wish them to use and hence the opportunity for creativity is diminished.

After looking at two, in my opinion good and complementary, models of the process of creativity, we need to think about the study of creativity. So far, the initial study was in the area of psychology and more recently a specificity towards Mathematics. In the psychological community, they have used six different approaches concerning the study of creativity (rather than the actual creative process) to try to understand it (Stenberg & Lubart 1996). These approaches are mystical, pragmatic, psychodynamic, psychometric, cognitive and social-personality I will briefly discuss them all as they will help us gain insight into how we can assess and develop creativity. As a teacher myself, the mystical approach of divine inspiration in the classroom might not be the best approach to helping a pupil become more creative. The pragmatic approach (developing without understanding it) can be seen as a development of the algorithmic process as the process involves using heuristics, some of which may have been not chosen if time was given to the problem. The psychodynamic approach, the tension between conscious reality and unconscious drivers chimes with the incubation stage of the Gestalt theory and as discussed earlier is reliant on time, a precious commodity in school life. The psychometric approach to studying creativity used paper and pencil methods to assess a subject’s creativity and we will discuss later this approach in more detail concerning Torrance testing. The cognitive approach “seeks to understand the mental representations and processes underlying creative thought” (Sternberg and Lubart, p 5 1998). In terms of Mathematics until the recent advent of computer graphical simulations that process could be said to be concerned with the images contained within the human mind and hence hard to assess in a fair manner. The final approach, social-personality brings in the importance of motivation, personality and sociocultural environment. With this last approach we can therefore say it is important to create the right environment for creativity to ensue for, as Sternberg and Lubart say, this occurs in a “supportive, evaluation-free environment” (p 6 1998). Overall, no single approach has been seen to be dominate when describing creativity but rather a “confluence of one or more of them” (Sriraman 2004).

If we look at two of the three most cited confluences of approach of studies, we can have insight into the environment that will help us to develop creativity in the classroom. When contemplating the system approach

given by Csikzentmihalyi, Sriraman (2004) talks of the time given to Mathematicians at University and the culture of inquisitiveness that is developed. We should cultivate a similar atmosphere in a classroom, although time constraints, curriculum requirements and occasionally competition between students may be a hindrance. In the system approach outlined by Csikzentmihalyi, creativity occurs in the interweaving of the individual, the field (the cultural organism) and the domain (the expert or critics). Hence, in a school we should foster a creative atmosphere by developing the equivalents, that being the individual, the programme of study and the class. Other theories of the confluence of the six approaches include the evolving systems of Grober and Wallace (1998) and the investment theory of Stenberg & Lubart (1996). The later paper once again stresses the importance of an environment that is both supportive and rewarding. The potential of transferring this environment to a classroom is obvious.

So now we have models of the process, the study of approach and the atmosphere needed to produce creativity in the classroom, can we attempt to assess it?

Assessing Creativity

Assuming we wish to assess on paper rather than merely observe creativity in the classroom, we must first discuss what is possible for a pupil to be able to achieve. If we are setting questions for a pupil then the production will either be towards a single solution (convergent thinking) or multiple solutions (divergent thinking) as defined by Guilford (1967). I would say that a pupil in a Mathematics classroom has been required to produce single solutions for most of their academic life so asking them to produce multiple solutions as a number of the creative assessments ask of them might well be an alien concept to them.

Secondly, we need to distinguish between the absolute and relativistic version of creativity in Mathematics. Absolute being the great works of creativity, (Andre Wiles and his proof of Fermat's Last Theorem could be an exemplary here) and relativistic which is what a pupil can be produce in a classroom. For this relativistic creativity it could be seen as an original solution when encountering a new topic or pulling in different strands together to give a fresh perspective on an old problem (Liljedahl & Sriraman 2006),

One of the six approaches to the study of creativity described earlier was psychometric which uses paper and pencil tasks to create divergent answers to a number of questions. The normally given example of this type of assessment are the Torrance tests (Torrance 1974) where the subject is asked a number of open-ended divergent questions. Answers are looked for that demonstrated fluency, flexibility, novelty and originality. Novelty was deemed the major link to creativity. Remember that these tests were to assess creativity in general and not specifically for Mathematics

For assessing creativity in Mathematics, questions in a similar vein have been developed. One approach developed by Leikin and her colleagues (Leikin 2009, Levav-Waynberg et al 2012, Leikin & Lev 2013) are Multiple Solution Tasks (MSTs) where pupils are asked to produce as many different types of solutions for the same question which has a convergent answer. A given example being solving a set of simultaneous equations. This might be strange for a pupil used to convergent thinking and I can imagine in a classroom the statement "I've got the answer why do I have to do it again" being used when first encountering them. With MSTs, pupils are assessed over three categories, fluency, flexibility and originality so similar to the Torrance test in terms of the psychometric approach although the pragmatic approach is also included due to the heuristic process. During a comparable time, Kwon et al (2006) created a comparable set of questions but in their case the solution space was divergent.

To understand how a MST assesses creativity I will give a brief overview of the process. Firstly, we must find possible different methods to solve the problem. Pupils are assessed against how many diverse methods they showed for their flexibility score. If a child uses two distinct approaches of Mathematics to produce their answer this will score higher than if they produce two variations from the same field. By referencing how many other students within their group utilised the same method their originality score is calculated. These values are then combined to give an overall creativity score with the highest values being produced with those sets of solutions which had the most flexible (varied) and original (rare in the group) solutions.

The fact originality is embedded in the scoring system ensures in my mind creativity is being assessed. The other aspect is that if a student produces one original method this will score higher than one producing a number of solutions that have a common method. Hence, in our assessment of pupil's creativity we can see not only who can attain the correct answers but also who can produce insightful answers from a variety of strands of Mathematics. As I mentioned when discussing the answer to the UKMT problem I wondered if one solution was more creative than another was. With the MST, it would be seen the one that was more original (i.e. less often submitted) was the more creative.

There are aspects of the use of MSTs that also interest me in terms of the implication of teaching with pupils for both myself and generally. If students realise that a good aspect of creativity in Mathematics is the ability of seeing a problem from different viewpoints then MSTs will help them to develop this skill. I have experienced before pupils who will ask if the topic being currently taught is linked to another a previous one. Even if this is not necessarily true in that instance, surely, the instinct that the pupil should try to link material from different areas is a good one and I should be encouraging them in this. This is generating what Liljedahl & Sriraman (2006) described as the "prepared mind". In addition, Krutetskii (1976) cited in Haylock (1987) stated that

"flexibility of mental processes" was an important component of mathematical ability in pupils and in the MST this is promoted.

With the use of MSTs and similar ideas, an actual score is produced but if we are to develop creativity in our pupils, we may also need to judge it on a more ad-hoc basis. One way is observing the process of creativity itself. This I believe can happen in two situations. Firstly in the classroom, where a pupil can bring in original thoughts and ideas as mentioned before. There is no finer moment for a teacher when the pupils in a classroom go "AHA" when they realise something after some thought. Here I have used the phrase from Liljedahl (2005) of that moment of creativity when something is realised.

The second place where I believe creativity can be observed is in the University interview process. If a candidate has been called for interview they may be asked to answer mathematical questions posed by the interviewer. Sadly, the time factor is a detriment to the incubation process. However, one piece of advice given to candidates is to talk through their thoughts so the interviewer has a chance to monitor the creative process in action, as described by Eryvnyck (1991). This may help to overcome a point made by Haylock that "It is, of course, the product of thinking which the teacher can observe, not the thinking process itself" (p 3 1987). Taken that Universities would like to recruit the most gifted and creative students you would say it is imperative that they include this type of assessment in the process.

Although originally from the initial task I wanted to answer if it was possible to assess creativity in Mathematics I now believe it is the development of creativity in the classroom that is more important for my teaching, and that the assessment will help us to achieve that. In Renzulli's (1986) three-ring model of giftedness, creativity is one of the said rings so needs to be developed in pupils. In terms of teaching generally, and more specifically mine, I now hope to incorporate the following ideas into my teaching after researching creativity.

Implications for Teaching

After looking at creativity, there are now a number of implications for my teaching. Not only to encourage and develop my pupils own creativity but also because, as Kattou et al (2013) suggest, "that the encouragement of mathematical creativity is important for further development of students' mathematical ability and understanding" (p 14). I would hope to incorporate the following.

Make certain pupils have the correct toolkit. Help them to develop an understanding of what the question is asking them and what knowledge they can already bring to the problem.

Give pupils time to reflect both in class and out. If it is a long and difficult problem this could be days but even in a classroom there should be no rush for instantaneous answers that would therefore be pushing simple algorithmic processes. If a problem is posed one day to be returned to later then the subconscious illumination

stage may be allowed to happen. Already I sometime allow this to happen in my lessons when I “plant some seeds” as I describe it.

Positively encourage pupils to bring in ideas and thoughts from other areas of Mathematics and see that it is not just a subject of separate rules and tools but has a beautiful interconnectedness.

Allow them and their peers to both express and review their own and others solutions to a problem. Let them be both the individual and domains in theirs and others creative process.

From first looking at the answer to the UKMT problem and being disappointed to now understanding more of the process and production of creativity, I have seen how it is important to develop creativity and assessment would give it prominence, to help us encourage more creative mathematicians in my teaching.

There is one note of caution about this field and that is how interconnected the people are in it. From the beginning with Torrance, Haylock, Ervynck and Sternburg & Lubart to more recently with Mann, Sriraman and Leikin et al they often cross-reference each other. However, I do believe their arguments are sound and that they are convincing on how we can define, assess and encourage creativity in Mathematics.

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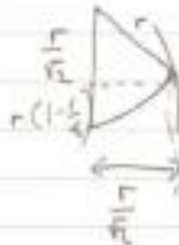
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Appendix

My Working for UKMT 2018 Question 25

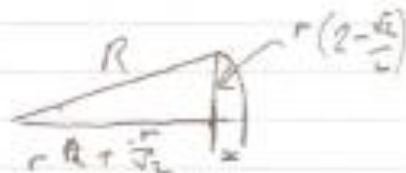
UKMT Qu25



$$\sin 45 = \frac{r}{r}$$

$$\frac{\sqrt{2} R^2}{4} = \frac{\sqrt{2} r^2}{2}$$

$$R^2 = 2r^2$$



$$r \left(1 - \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}-1}{\sqrt{2}}\right) r = \frac{2-\sqrt{2}}{2}$$

$$\therefore R^2 = r^2 \left(1 + \frac{1}{\sqrt{2}}\right)^2 + r^2 \left(1 - \frac{1}{\sqrt{2}}\right)^2$$

$$R^2 = r^2 \left(1 + \frac{2}{\sqrt{2}} + \frac{1}{2}\right) + \left(1 - \frac{2}{\sqrt{2}} + \frac{1}{2}\right)$$

$$R^2 = 3r^2$$

$$\frac{\frac{1}{2} \sqrt{2} r^2}{\frac{1}{4} \sqrt{2} R^2} = \frac{\frac{1}{2} \pi 3r^2}{\frac{1}{4} \pi 3r^2} = \frac{2}{3}$$



United Kingdom
Mathematics Trust

SENIOR MATHEMATICAL CHALLENGE

Tuesday 6th November 2018

Organised by the United Kingdom Mathematics Trust



Candidates must be full-time students at secondary school or FE college.

England & Wales: Year 13 or below

Scotland: S6 or below

Northern Ireland: Year 14 or below

INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**.
No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.
4. Use a **B or an HB non-propelling pencil**. Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed**. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:** all candidates start with 25 marks;
0 marks are awarded for each question left unanswered;
4 marks are awarded for each correct answer;
1 mark is deducted for each incorrect answer (to discourage guessing).
7. Your Answer Sheet will be read by a machine. **Do not write or doodle on the sheet except to mark your chosen options**. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way.
8. **The questions on this paper are designed to challenge you to think, not to guess**. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Senior Mathematical Challenge should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

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